

MONOPOLE PROBLEM AND EXTENSIONS OF SUPERSYMMETRIC HYBRID INFLATION

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We discuss, in the context of a concrete supersymmetric grand unified model based on the Pati-Salam gauge group $SU(4)_c \times SU(2)_L \times SU(2)_R$, two ‘natural’ extensions of supersymmetric hybrid inflation, which avoid the cosmological disaster encountered in the standard hybrid inflationary scenario from the overproduction of monopoles at the end of inflation. Successful ‘reheating’ which satisfies the gravitino constraint takes place after the end of inflation. Also, adequate baryogenesis via a primordial leptogenesis occurs consistently with the solar and atmospheric neutrino oscillation data as well as the $SU(4)_c$ symmetry. Moreover, the μ -term is generated via a Peccei-Quinn symmetry and proton is practically stable.

1 Introduction

Inflation offers an elegant solution to the outstanding problems of the standard Big-Bang cosmological model and predicts the formation of the large scale structure of the universe and the temperature fluctuations which are observed in the cosmic microwave background radiation (CMBR). It also solves the cosmological problem caused by the overproduction of grand unified theory (GUT) magnetic monopoles as well as other unwanted relics such as domain walls, gravitini or moduli fields.

However, the early realizations of inflation require extremely flat potentials and very small coupling constants. To solve this naturalness problem, the hybrid inflationary scenario has been introduced [1]. The basic idea was to use two real scalar fields χ and σ instead of one that was normally used. The field χ may be a gauge non-singlet and provides the ‘vacuum’ energy density which drives inflation, while σ is the slowly varying field during inflation. This splitting of roles between two fields allows us to reproduce the observed temperature fluctuations of the CMBR with ‘natural’ (not too small) values of the relevant parameters in contrast to previous realizations of inflation.

The scalar potential for hybrid inflation possesses a valley of local minima with respect to χ with large ‘vacuum’ energy density. This valley lies at $\chi = 0$

with σ being greater than a certain critical (instability) value σ_c , and has a classical inclination provided by the mass of σ . The global minima of the potential lie at $\chi \neq 0$ and $\sigma = 0$. As the system rolls down the valley of local minima, the slow-roll conditions (see e.g., Ref.[2]) are satisfied and inflation takes place. Inflation ends abruptly as σ falls below σ_c . It is followed by a ‘waterfall’ regime and χ starts oscillating about a global minimum of the potential acquiring a non-vanishing vacuum expectation value (vev). If χ is a gauge non-singlet, spontaneous gauge symmetry breaking occurs at the end of inflation, and topological defect can potentially form [3].

The simplest framework for realizing hybrid inflation is provided [4,5] by supersymmetric (SUSY) GUTs which are based on gauge groups with rank greater than five. The same superpotential which lowers the rank of the gauge group also leads [5] to successful hybrid inflation with ‘natural’ values of the relevant parameter and a gauge symmetry breaking scale of the order of the SUSY GUT scale. The slowly rolling inflaton field belongs to a gauge singlet superfield which couples to a conjugate pair of gauge non-singlet Higgs superfields. The tree-level scalar potential possesses a flat valley of local minima for values of the gauge singlet inflaton greater than a certain critical value. Along this valley, the vevs of the Higgs superfields vanish, there exists a constant non-zero ‘vacuum’ energy density and SUSY is broken. The (classical) flatness of the valley is lifted by the one-loop radiative corrections [5] to the scalar potential which are calculated with the GUT gauge symmetry being restored and SUSY being broken. A variant of Linde’s scenario is thus obtained. Inflation ends by a ‘waterfall’ regime as the gauge singlet falls below its critical value, the Higgs fields and the gauge singlet start oscillating about the SUSY minima of the potential where the Higgs vevs are non-zero.

If the SUSY vacuum manifold is homotopically non-trivial, topological defects will be copiously formed [3] by the Kibble mechanism [6] since the system can end up at any point of the vacuum manifold with equal probability. So a cosmological disaster is encountered in the hybrid inflationary models which are based on a gauge symmetry breaking which predicts the existence of magnetic monopoles. One way out of this catastrophe is to do this symmetry breaking in two steps by introducing an intermediate symmetry breaking scale between the GUT and the standard model scales. The intermediate gauge symmetry must be chosen such that the unwanted monopoles are formed in the first step of symmetry breaking, and hybrid inflation occurs in the second step which does not lead to the formation of new unwanted topological defects. Inflation then dilutes the pre-existing monopoles without generating new ones. The rank of the gauge group must be lowered in the second step and, in many realistic GUTs, cosmic strings are formed at the end of inflation [7]. They will contribute to the CMBR anisotropy in a proportion which depends upon the GUT gauge group and the cosmic microwave explorer (COBE) [8] normalization for strings and inflation.

One idea [3,9,10] for solving the monopole problem of hybrid inflation is

to include into the standard superpotential for hybrid inflation the leading non-renormalizable term. This term, as we will explain in the next section, cannot be excluded by any symmetries and, if its dimensionless coefficient is of order unity, can be comparable with the trilinear coupling of the standard superpotential (whose coefficient is $\sim 10^{-3}$). Actually, we have two options. We can either keep [9] both these terms or remove [3,10] the trilinear term by imposing an appropriate discrete symmetry and keep only the leading non-renormalizable term. The pictures which emerge in the two cases are quite different. However, they share an important common feature. The GUT gauge group is already broken during inflation and thus no topological defects can form at the end of inflation. Consequently, the monopole problem is solved even in GUTs with a single step of symmetry breaking.

Furthermore, the constraints on the quadrupole anisotropy of the CMBR from the COBE [8] measurements can be easily satisfied. Our model possesses a number of other interesting features too. The μ problem of the minimal supersymmetric standard model (MSSM) is solved [11] via a Peccei-Quinn (PQ) symmetry [12] which also solves the strong CP problem. Although the baryon and lepton numbers are explicitly violated, the proton life time is considerably higher than the present experimental limits. Light neutrinos acquire hierarchical masses by the seesaw mechanism and the baryon asymmetry of the universe (BAU) can be generated via a primordial leptogenesis [13] (for a recent review see Ref.[14]). The gravitino constraint [15] on the ‘reheat’ temperature, the low deuterium abundance limits [16] on the BAU and the requirement of almost maximal $\nu_\mu - \nu_\tau$ mixing from SuperKamiokande [17] can be met for μ - and τ -neutrino masses restricted by the small or large mixing angle MSW solution [18] of the solar neutrino puzzle and SuperKamiokande respectively. The required values of the relevant parameters are ‘natural’.

2 SUSY Hybrid Inflation and its Extensions

We will now summarize the standard SUSY hybrid inflationary scenario in the context of a concrete SUSY GUT and discuss its extensions which solve the magnetic monopole problem encountered in the standard scenario. Along the lines of Refs.[9,10], we consider the SUSY Pati-Salam (PS) model [19] which is one of the simplest GUT models predicting magnetic monopoles. This model is based on the PS gauge group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$. The PS monopoles carry two units of ‘Dirac’ magnetic charge [20]. We will present possible solutions of the magnetic monopole problem of hybrid inflation within the SUSY PS model [9,10]. It is worth mentioning, however, that these solutions can be readily applied to other semi-simple gauge groups too such as the ‘trinification’ group $SU(3)_c \times SU(3)_L \times SU(3)_R$, which emerges from string theory and predicts [21] monopoles with triple ‘Dirac’ magnetic charge, and possibly to simple gauge groups such as $SO(10)$.

In the SUSY PS model, the left-handed quark and lepton superfields are

accommodated in the following representations:

$$\begin{aligned} F_i &= (4, 2, 1) \equiv \begin{pmatrix} u_i & u_i & u_i & \nu_i \\ d_i & d_i & d_i & e_i \end{pmatrix}, \\ F_i^c &= (\bar{4}, 1, 2) \equiv \begin{pmatrix} u_i^c & u_i^c & u_i^c & \nu_i^c \\ d_i^c & d_i^c & d_i^c & e_i^c \end{pmatrix}, \end{aligned} \quad (1)$$

where the subscript $i = 1, 2, 3$ denotes the family index. The G_{PS} gauge symmetry can be spontaneously broken to the standard model gauge group by a pair of Higgs superfields

$$\begin{aligned} H^c &= (\bar{4}, 1, 2) \equiv \begin{pmatrix} u_H^c & u_H^c & u_H^c & \nu_H^c \\ d_H^c & d_H^c & d_H^c & e_H^c \end{pmatrix}, \\ \bar{H}^c &= (4, 1, 2) \equiv \begin{pmatrix} \bar{u}_H^c & \bar{u}_H^c & \bar{u}_H^c & \bar{\nu}_H^c \\ \bar{d}_H^c & \bar{d}_H^c & \bar{d}_H^c & \bar{e}_H^c \end{pmatrix} \end{aligned} \quad (2)$$

acquiring non-vanishing vevs in the right-handed neutrino direction, $\langle \nu_H^c \rangle$, $\langle \bar{\nu}_H^c \rangle \neq 0$. The two low energy Higgs doublets of the MSSM are contained in the following representation:

$$h = (1, 2, 2) \equiv \begin{pmatrix} h_2^+ & h_1^0 \\ h_2^0 & h_1^- \end{pmatrix}. \quad (3)$$

After the breaking of G_{PS} , the bidoublet Higgs field h splits into two Higgs doublets h_1 , h_2 , whose neutral components subsequently develop weak vevs $\langle h_1^0 \rangle = v_1$ and $\langle h_2^0 \rangle = v_2$ with $\tan \beta = v_2/v_1$.

The (renormalizable) superpotential for the breaking of G_{PS} is

$$W = \kappa S(-M^2 + H^c \bar{H}^c), \quad (4)$$

where S is a gauge singlet left-handed superfield and the parameters κ , M can be made positive by field redefinitions. The vanishing of the F-term F_S implies that $\langle H^c \rangle \langle \bar{H}^c \rangle = M^2$, whereas the D-terms vanish for $|\langle H^c \rangle| = |\langle \bar{H}^c \rangle|$. So, the SUSY vacua (rotated to the real axis) lie at $\langle H^c \rangle = \langle \bar{H}^c \rangle^* = \pm M$ and $\langle S \rangle = 0$ (from $F_{H^c} = F_{\bar{H}^c} = 0$). We see that W leads to the spontaneous breaking of G_{PS} .

It is interesting to note that the same superpotential which breaks G_{PS} also leads to hybrid inflation. The potential derived from W in Eq.(4) is

$$V(H^c, \bar{H}^c, S) = \kappa^2 |M^2 - H^c \bar{H}^c|^2 + \kappa^2 |S|^2 (|H^c|^2 + |\bar{H}^c|^2) + \text{D-terms}. \quad (5)$$

For $|S| > S_c \equiv M$, the potential V is minimized by $H^c = \bar{H}^c = 0$. This yields a classically flat valley of local minima. However, the flatness of this valley is lifted at the one-loop level. The SUSY breaking by the ‘vacuum’ energy density $\kappa^2 M^4$ along this valley causes a mass splitting in the supermultiplets H^c , \bar{H}^c . We obtain a Dirac fermion with mass² equal to $\kappa^2 |S|^2$ and two complex scalars with mass² equal to $\kappa^2 |S|^2 \pm \kappa^2 M^2$. This leads to the existence

of important one-loop radiative corrections to V on the valley which can be found from the Coleman-Weinberg formula [22]:

$$\Delta V = \frac{1}{64\pi^2} \sum_i (-)^{F_i} M_i^4 \ln \frac{M_i^2}{\Lambda^2}, \quad (6)$$

where the sum extends over all helicity states i , F_i and M_i^2 are the fermion number and mass² of the i th state, and Λ is a renormalization mass scale. We find that $\Delta V(|S|)$ is given [23] by

$$\kappa^2 M^4 \frac{\kappa^2}{4\pi^2} \left(2 \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right), \quad (7)$$

where $z = |S|^2/M^2$. For $z \gg 1$ ($|S| \gg S_c$), the effective potential on the valley can be expanded [5,24] as

$$V_{\text{eff}}(|S|) = \kappa^2 M^4 \left[1 + \frac{\kappa^2}{2\pi^2} \left(\ln \frac{\kappa^2 |S|^2}{\Lambda^2} + \frac{3}{2} - \frac{1}{12z^2} + \dots \right) \right]. \quad (8)$$

We see that the one-loop radiative corrections generate a (Λ -independent) slope along the classically flat valley of local minima. So this valley can, in principle, be used as an inflationary trajectory. As the system rolls down the valley driven by the contribution in Eq.(8), the energy density is dominated by the tree-level ‘vacuum’ energy density $\kappa^2 M^4$, the slow-roll conditions hold, and inflation takes place till $|S|$ reaches its critical value S_c . The COBE [8] measurements on the quadrupole anisotropy of the CMBR can be reproduced [5] with ‘natural’ values of κ , and M ’s close to the SUSY GUT scale.

At S_c , the system enters into a ‘waterfall’ regime followed by damped oscillations about the SUSY vacua where H^c and \bar{H}^c acquire non-zero vevs and G_{PS} breaks. It is an important feature of the scenario that the G_{PS} gauge symmetry is restored along the inflationary trajectory and breaks spontaneously only at the end of inflation when the system falls towards the SUSY minima. This transition then leads [3] to a cosmologically unacceptable copious production of doubly charged magnetic monopoles. One way to resolve this problem, which arises if standard hybrid inflation is employed, is to use as inflationary trajectory another flat direction in which G_{PS} is already broken. Such a direction naturally appears if we include the next order non-renormalizable superpotential coupling of S to H^c , \bar{H}^c . The trilinear term in Eq.(4) can be either kept [9] or removed [3,10] by a discrete symmetry.

2.1 Shifted Hybrid Inflation

As mentioned above, the cosmological monopole problem can be solved by including the leading non-renormalizable term in the superpotential for hybrid inflation. We will first examine the case where the trilinear term in Eq.(4) is also kept. The coexistence of these terms leads [9] to the appearance of a new ‘shifted’ classically flat direction where the G_{PS} gauge symmetry is broken,

i.e., the Higgs fields H^c , \bar{H}^c possess (constant) non-vanishing vevs. The trivial valley of minima where G_{PS} is restored is also present. The ‘shifted’ flat direction can be used as an alternative inflationary trajectory with the necessary inclination obtained again from one-loop radiative corrections, which now have to be calculated with both the GUT gauge symmetry and SUSY being broken. The termination of inflation is again abrupt followed by a ‘waterfall’, but no monopoles are formed in this transition since G_{PS} is already spontaneously broken during inflation.

The relevant part of the superpotential, which includes the leading non-renormalizable term, is

$$W = \kappa S(-M^2 + H^c \bar{H}^c) - \beta \frac{S(H^c \bar{H}^c)^2}{M_S^2}, \quad (9)$$

where $M_S \approx 5 \times 10^{17}$ GeV is the string scale and β is taken positive for simplicity. D-flatness implies that $H^{c*} = e^{i\theta} \bar{H}^c$. We restrict ourselves to the direction with $\theta = 0$ ($H^{c*} = \bar{H}^c$) containing the non-trivial (‘shifted’) inflationary path (see below). The scalar potential derived from W in Eq.(9) then takes the form

$$V = \left[\kappa(|H^c|^2 - M^2) - \beta \frac{|H^c|^4}{M_S^2} \right]^2 + 2\kappa^2 |S|^2 |H^c|^2 \left[1 - \frac{2\beta}{\kappa M_S^2} |H^c|^2 \right]^2. \quad (10)$$

Defining the dimensionless variables $y = |H^c|/M$, $w = |S|/M$, we obtain

$$\tilde{V} = \frac{V}{\kappa^2 M^4} = (y^2 - 1 - \xi y^4)^2 + 2w^2 y^2 (1 - 2\xi y^2)^2, \quad (11)$$

where $\xi = \beta M^2 / \kappa M_S^2$. This potential is a simple extension of the standard potential for SUSY hybrid inflation (which corresponds to $\xi = 0$) and appears in a wide class of models incorporating the leading non-renormalizable correction to the standard hybrid inflationary superpotential.

For constant w (or $|S|$), \tilde{V} in Eq.(11) has extrema at

$$y_1 = 0, \quad y_2 = \frac{1}{\sqrt{2\xi}}, \quad y_{3\pm} = \frac{1}{\sqrt{2\xi}} \sqrt{(1 - 6\xi w^2) \pm \sqrt{(1 - 6\xi w^2)^2 - 4\xi(1 - w^2)}}. \quad (12)$$

Note that the first two extrema (at y_1, y_2) are $|S|$ -independent and, thus, correspond to classically flat directions, the trivial one at $y_1 = 0$ with $\tilde{V}_1 = 1$, and the non-trivial one at $y_2 = 1/\sqrt{2\xi} = \text{constant}$ with $\tilde{V}_2 = (1/4\xi - 1)^2$, which we will use as our inflationary path. The trivial trajectory is a valley of minima for $w > 1$, while the non-trivial one for $w > w_0 = (1/8\xi - 1/2)^{1/2}$, which is its instability (critical) point. We take $\xi < 1/4$, so that $w_0 > 0$ and the non-trivial path is destabilized before w reaches zero (the destabilization is in the chosen direction $H^{c*} = \bar{H}^c$). The extrema at $y_{3\pm}$, which are $|S|$ -dependent and non-flat, do not exist for all values of w and ξ , since the expressions under the square roots in Eq.(12) are not always non-negative. These two extrema, at $w = 0$, become the SUSY vacua. The relevant SUSY

vacuum (see below) corresponds to $y_{3-}(w=0)$ and, thus, the absolute value v_0 of the common vev of H^c *, \bar{H}^c is given by

$$\left(\frac{v_0}{M}\right)^2 = \frac{1}{2\xi}(1 - \sqrt{1 - 4\xi}). \quad (13)$$

We will now discuss the structure of \tilde{V} and the inflationary history in the most interesting range of ξ , which is $1/4 > \xi > 1/6$. For fixed $w > 1$, there exist two local minima at $y_1 = 0$ and $y_2 = 1/\sqrt{2\xi}$, which corresponds to lower potential energy density, and a local maximum at y_{3+} lying between the minima. As w becomes smaller than unity, the extremum at y_1 turns into a local maximum, while the extremum at y_{3+} disappears. The system can freely fall into the non-trivial (desirable) trajectory at y_2 even if it started at $y_1 = 0$. As we further decrease w below $(2 - \sqrt{36\xi - 5})^{1/2}/3\sqrt{2\xi}$, a pair of new extrema, a local minimum at y_{3-} and a local maximum at y_{3+} , are created between y_1 and y_2 . As w crosses $(1/8\xi - 1/2)^{1/2}$, the local maximum at y_{3+} crosses y_2 becoming a local minimum. At the same time, the local minimum at y_2 turns into a local maximum and inflation along the ‘shifted’ trajectory is terminated with the system falling into the local minimum at y_{3-} which, at $w = 0$, develops into a SUSY vacuum.

We see that, no matter where the system starts from, it always passes from the ‘shifted’ trajectory, where the relevant part of inflation takes place, before falling into the SUSY vacuum. So, G_{PS} is already broken during inflation and no monopoles are produced at the ‘waterfall’.

The COBE [8] result can be reproduced, for instance, with $\kappa \approx 4 \times 10^{-3}$, which corresponds to $\xi = 1/5$, $v_0 \approx 1.7 \times 10^{16}$ GeV, $M \approx 1.45 \times 10^{16}$ GeV (for $\beta = 1$, $M_S = 5 \times 10^{17}$ GeV). Notice that $v_0 \sim 10^{16}$ GeV consistently with the unification of the MSSM gauge couplings. The spectral index $n = 0.954$.

After inflation, the system could possibly fall into the minimum at y_{3+} . This, however, does not happen since in the last e-folding or so the barrier between the minima at y_{3-} and y_2 is considerably reduced and the decay of the ‘false vacuum’ at y_2 to the minimum at y_{3-} is completed within a fraction of an e-folding before the y_{3+} minimum even comes into existence.

2.2 Smooth Hybrid Inflation

An alternative solution [3,10] to the monopole problem of hybrid inflation can be constructed by imposing, in the model of Sec.2.1, an extra Z_2 symmetry under which $H^c \bar{H}^c \rightarrow -H^c \bar{H}^c$ (say $H^c \rightarrow -H^c$). The whole structure of the model remains unaltered except that now only even powers of the combination $H^c \bar{H}^c$ are allowed in the superpotential terms.

The inflationary superpotential in Eq.(9) becomes

$$W = S \left(-\mu^2 + \frac{(H^c \bar{H}^c)^2}{M_S^2} \right), \quad (14)$$

where we absorbed the dimensionless parameters κ, β in μ, M_S . The resulting scalar potential V is then given by

$$\tilde{V} = \frac{V}{\mu^4} = (1 - \tilde{\chi}^4)^2 + 16\tilde{\sigma}^2\tilde{\chi}^6, \quad (15)$$

where we used the dimensionless fields $\tilde{\chi} = \chi/2(\mu M_S)^{1/2}$, $\tilde{\sigma} = \sigma/2(\mu M_S)^{1/2}$ with χ, σ being normalized real scalar fields defined by $\nu_H^c = \bar{\nu}_H^c = \chi/2$, $S = \sigma/\sqrt{2}$ after rotating $\nu_H^c, \bar{\nu}_H^c, S$ to the real axis.

The emerging picture is completely different. The flat direction at $\tilde{\chi} = 0$ is now a local maximum with respect to $\tilde{\chi}$ for all values of $\tilde{\sigma}$, and two new symmetric valleys of minima appear [3,10] at

$$\tilde{\chi} = \pm\sqrt{6}\tilde{\sigma} \left[\left(1 + \frac{1}{36\tilde{\sigma}^4} \right)^{\frac{1}{2}} - 1 \right]^{\frac{1}{2}}. \quad (16)$$

They contain the SUSY vacua which lie at $\tilde{\chi} = \pm 1, \tilde{\sigma} = 0$. Note that these valleys are not classically flat. In fact, they possess an inclination already at the classical level, which can drive the inflaton towards the vacua. As a consequence, contrary to the case of standard SUSY or shifted hybrid inflation, there is no need of radiative corrections, which are expected to give a subdominant contribution to the slope of the inflationary paths. In spite of this, one could try to include the one-loop corrections. This requires the construction of the mass spectrum on the inflationary trajectories. In doing so, we find that the mass² of some scalars belonging to the inflaton sector is negative. The one-loop corrections, which involve logarithms of the masses squared, are then ill-defined. This may be remedied by resumming the perturbative expansion to all orders, which is a formidable task and we do not pursue it here.

The potential along the symmetric valleys of minima is given by [3,10]

$$\begin{aligned} \tilde{V} &= 48\tilde{\sigma}^4 \left[72\tilde{\sigma}^4 \left(1 + \frac{1}{36\tilde{\sigma}^4} \right) \left(\left(1 + \frac{1}{36\tilde{\sigma}^4} \right)^{\frac{1}{2}} - 1 \right) - 1 \right] \\ &= 1 - \frac{1}{216\tilde{\sigma}^4} + \dots, \quad \text{for } \tilde{\sigma} \gg 1. \end{aligned} \quad (17)$$

The system follows, from the beginning, a particular inflationary trajectory and, thus, ends up at a particular point of the vacuum manifold leading to no production of disastrous magnetic monopoles.

Inflation does not come to an abrupt end in this case since the inflationary path is stable with respect to $\tilde{\chi}$ for all $\tilde{\sigma}$'s. The value $\tilde{\sigma}_0$ of $\tilde{\sigma}$ at which inflation is terminated smoothly is found from the ϵ and η criteria (see e.g., Ref.[2]), and the derivatives [10] of the potential along the inflationary path:

$$\frac{d\tilde{V}}{d\tilde{\sigma}} = 192\tilde{\sigma}^3 \left[(1 + 144\tilde{\sigma}^4) \left(\left(1 + \frac{1}{36\tilde{\sigma}^4} \right)^{\frac{1}{2}} - 1 \right) - 2 \right], \quad (18)$$

$$\begin{aligned} \frac{d^2 \tilde{V}}{d\tilde{\sigma}^2} = \frac{16}{3\tilde{\sigma}^2} & \left\{ (1 + 504\tilde{\sigma}^4) \left[72\tilde{\sigma}^4 \left(\left(1 + \frac{1}{36\tilde{\sigma}^4} \right)^{\frac{1}{2}} - 1 \right) - 1 \right] \right. \\ & \left. - (1 + 252\tilde{\sigma}^4) \left(\left(1 + \frac{1}{36\tilde{\sigma}^4} \right)^{-\frac{1}{2}} - 1 \right) \right\}. \end{aligned} \quad (19)$$

Here, we have the freedom to identify the vev $v_0 = |\langle H^c \rangle| = |\langle \bar{H}^c \rangle|$, which equals $(\mu M_S)^{1/2}$, with the SUSY GUT scale $M_G \approx 2.86 \times 10^{16}$ GeV. From COBE [8], we then obtain $M_S \approx 4.39 \times 10^{17}$ GeV and $\mu \approx 1.86 \times 10^{15}$ GeV.

3 Relevant Phenomenological and Cosmological Constraints: Shifted versus Smooth Hybrid Inflation

3.1 The μ Problem

An important shortcoming of MSSM is that there is no understanding of how the SUSY μ -term, with the right magnitude of $|\mu| \sim 10^2 - 10^3$ GeV, arises. In both scenarios of shifted and smooth hybrid inflation, one way [11] to solve this μ problem is via a PQ symmetry $U(1)_{PQ}$ [12], which also solves the strong CP problem. This solution is based on the observation [25] that the axion decay constant f_a , which is the symmetry breaking scale of $U(1)_{PQ}$, is (normally) ‘intermediate’ ($\sim 10^{11} - 10^{12}$ GeV) and, thus, $|\mu| \sim f_a^2/M_S$. The scale f_a is, in turn, $\sim (m_{3/2} M_S)^{1/2}$, where $m_{3/2} \sim 1$ TeV is the gravity-mediated soft SUSY breaking scale (gravitino mass). In order to implement this solution of the μ problem, we introduce a pair of gauge singlet superfields N, \bar{N} with PQ charges -1, 1 and the non-renormalizable couplings $\lambda_1 N^2 h^2/M_S, \lambda_2 N^2 \bar{N}^2/M_S$ in the superpotential. Here, $\lambda_{1,2}$ are taken positive by redefining the phases of N, \bar{N} . After SUSY breaking, the $N^2 \bar{N}^2$ term leads to the scalar potential:

$$\begin{aligned} V_{PQ} = & \left(m_{3/2}^2 + 4\lambda_2^2 \left| \frac{N\bar{N}}{M_S} \right|^2 \right) [(|N| - |\bar{N}|)^2 + 2|N||\bar{N}|] \\ & + 2|A|m_{3/2}\lambda_2 \frac{|N\bar{N}|^2}{M_S} \cos(\epsilon + 2\theta + 2\bar{\theta}), \end{aligned} \quad (20)$$

where A is the dimensionless coefficient of the soft SUSY breaking term corresponding to the superpotential term $N^2 \bar{N}^2$ and $\epsilon, \theta, \bar{\theta}$ are the phases of A, N, \bar{N} respectively. Minimization of V_{PQ} then requires $|N| = |\bar{N}|$, $\epsilon + 2\theta + 2\bar{\theta} = \pi$ and V_{PQ} takes the form

$$V_{PQ} = 2|N|^2 m_{3/2}^2 \left(4\lambda_2^2 \frac{|N|^4}{m_{3/2}^2 M_S^2} - |A|\lambda_2 \frac{|N|^2}{m_{3/2} M_S} + 1 \right). \quad (21)$$

For $|A| > 4$, the absolute minimum of the potential is at

$$|\langle N \rangle| = |\langle \bar{N} \rangle| \equiv \frac{f_a}{2} = (m_{3/2} M_S)^{\frac{1}{2}} \left(\frac{|A| + (|A|^2 - 12)^{\frac{1}{2}}}{12\lambda_2} \right)^{\frac{1}{2}} \sim (m_{3/2} M_S)^{\frac{1}{2}}. \quad (22)$$

The μ -term is generated via the $N^2 h^2$ superpotential term with $|\mu| = 2\lambda_1 |\langle N \rangle|^2 / M_S$, which is of the right magnitude.

The potential V_{PQ} also has a local minimum at $N = \bar{N} = 0$, which is separated from the global PQ minimum by a sizable potential barrier preventing a successful transition from the trivial to the PQ vacuum. This situation persists at all cosmic temperatures after the ‘reheating’ which follows hybrid inflation, as has been shown [9] by considering the one-loop temperature corrections [26] to the potential. We are, thus, obliged to assume that, after the end of inflation, the system emerges in the PQ vacuum since, otherwise, it will be stuck for ever in the trivial vacuum.

3.2 ‘Reheating’ and Leptogenesis

A complete inflationary scenario should be followed by a successful ‘reheating’ satisfying the gravitino constraint [15] on the ‘reheat’ temperature, $T_r \lesssim 10^9$ GeV, and generating the observed BAU. After the end of inflation, the system falls towards the SUSY vacuum and performs damped oscillations about it. The inflaton (oscillating system) consists of the two complex scalar fields $\theta = (\delta\nu_H^c + \delta\bar{\nu}_H^c)/\sqrt{2}$ ($\delta\nu_H^c = \nu_H^c - v_0$, $\delta\bar{\nu}_H^c = \bar{\nu}_H^c - v_0$) and S , with equal mass $m_{\text{infl}} = \sqrt{2}\kappa v_0(1 - 2\xi v_0^2/M^2)$ or $2\sqrt{2}(\mu/M_S)^{1/2}\mu$ for shifted or smooth hybrid inflation respectively.

The fields θ and S decay into a pair of right-handed neutrinos ($\psi_{\nu_i^c}$) and sneutrinos (ν_i^c) respectively via the coupling $\gamma_i \bar{H}^c \bar{H}^c F_i^c F_i^c / M_S$ and the terms in Eq.(9) or (14) in the shifted or smooth case. The Lagrangian terms are:

$$L_{\text{decay}}^\theta = -\sqrt{2}\gamma_i \frac{v_0}{M_S} \theta \psi_{\nu_i^c} \psi_{\nu_i^c} + h.c. , \quad (23)$$

$$L_{\text{decay}}^S = -\sqrt{2}\gamma_i \frac{v_0}{M_S} S^* \nu_i^c \nu_i^c m_{\text{infl}} + h.c. , \quad (24)$$

and the common, as it turns out, decay width is given by

$$\Gamma = \Gamma_{\theta \rightarrow \bar{\psi}_{\nu_i^c} \bar{\psi}_{\nu_i^c}} = \Gamma_{S \rightarrow \nu_i^c \nu_i^c} = \frac{1}{8\pi} \left(\frac{M_i}{v_0} \right)^2 m_{\text{infl}} , \quad (25)$$

provided that the mass $M_i = 2\gamma_i v_0^2 / M_S$ of the relevant ν_i^c satisfies the inequality $M_i < m_{\text{infl}}/2$. The same number of particles and sparticles is produced after inflation, and thus the SUSY world is recovered.

To minimize the number of small coupling constants, we assume that

$$M_2 < \frac{1}{2} m_{\text{infl}} \leq M_3 , \quad (26)$$

so that the coupling γ_3 can be of order unity. The inflaton then decays into the second heaviest right-handed neutrino superfield with mass M_2 . Note that there always exist γ_3 's smaller than unity such that the second inequality in Eq.(26) is satisfied for all relevant values of the other parameters.

The ‘reheat’ temperature T_r , for the MSSM spectrum, is given [24] by

$$T_r \approx \frac{1}{7}(\Gamma M_P)^{\frac{1}{2}}, \quad (27)$$

and must satisfy the gravitino constraint [15], $T_r \lesssim 10^9$ GeV, for gravity-mediated SUSY breaking with universal boundary conditions. To maximize the naturalness of the model, we take the maximal M_2 (and thus γ_2) allowed by the gravitino constraint. These M_2 's turn out to be much smaller than the values of $m_{\text{infl}}/2$ and, thus, the first inequality in Eq.(26) is well satisfied.

Another important constraint comes from the BAU. In this model, a primordial lepton asymmetry [13] is produced which is then partly converted into baryon asymmetry by the non-perturbative electroweak sphaleron effects [27]. Actually, in the PS model under consideration as well as in many other models, this is the only way to generate the observed BAU since the inflaton decays into right-handed neutrino superfields. The subsequent decay of these superfields into lepton (antilepton) L (\bar{L}) and electroweak Higgs superfields can only produce a lepton asymmetry. It is important to ensure that this lepton asymmetry is not erased [28] by lepton number violating $2 \rightarrow 2$ scattering processes such as $LL \rightarrow h_2^* h_2^*$ or $Lh_2 \rightarrow \bar{L}h_2^*$ at all temperatures between T_r and 100 GeV. This is automatically satisfied since the lepton asymmetry is protected [29] by SUSY at temperatures between T_r and $T \sim 10^7$ GeV and, for $T \lesssim 10^7$ GeV, these scattering processes are well out of equilibrium provided [29] $m_{\nu_\tau} \lesssim 10$ eV, which readily holds in our case (see below). For MSSM spectrum, the observed BAU n_B/s is related [29] to the primordial lepton asymmetry n_L/s by $n_B/s = (-28/79)n_L/s$. Thus, the low deuterium abundance constraint [16] on the BAU gives $1.8 \times 10^{-10} \lesssim -n_L/s \lesssim 2.3 \times 10^{-10}$.

As already mentioned, the lepton asymmetry is produced through the decay of the superfield ν_2^c , which emerges as decay product of the inflaton. This superfield decays into electroweak Higgs and (anti)lepton superfields. The relevant one-loop diagrams are both of the vertex and self-energy type [30] with an exchange of ν_3^c . The resulting lepton asymmetry is [31]

$$\frac{n_L}{s} \approx 1.33 \frac{9T_r}{16\pi m_{\text{infl}}} \frac{M_2}{M_3} \frac{c^2 s^2 \sin 2\delta (m_3^{D\,2} - m_2^{D\,2})^2}{|\langle h_2 \rangle|^2 (m_3^{D\,2} s^2 + m_2^{D\,2} c^2)}, \quad (28)$$

where $|\langle h_2 \rangle| \approx 174$ GeV, $m_{2,3}^D$ ($m_2^D \leq m_3^D$) are the ‘Dirac’ neutrino masses (in a basis where they are diagonal and positive), and $c = \cos \theta$, $s = \sin \theta$, with θ and δ being the rotation angle and phase which diagonalize the Majorana mass matrix of the right-handed neutrinos. Note that Eq.(28) holds [32] provided that $M_2 \ll M_3$ and the decay width of ν_3^c is $\ll (M_3^2 - M_2^2)/M_2$, and both conditions are well satisfied in our model. Here, we concentrated on the two

heaviest families ($i = 2, 3$) and ignored the first one. We were able to do this since the analysis [33] of the CHOOZ experiment [34] shows that the solar and atmospheric neutrino oscillations decouple.

The light neutrino mass matrix is given by the seesaw formula:

$$m_\nu \approx -\tilde{m}^D \frac{1}{M} m^D, \quad (29)$$

where m^D is the ‘Dirac’ neutrino mass matrix and M the Majorana mass matrix of right-handed neutrinos. The determinant and the trace invariance of $m_\nu^\dagger m_\nu$ imply [31] two constraints on the asymptotic (at M_G) parameters which take the form:

$$m_2 m_3 = \frac{(m_2^D m_3^D)^2}{M_2 M_3}, \quad (30)$$

$$m_2^2 + m_3^2 = \frac{(m_2^{D^2} c^2 + m_3^{D^2} s^2)^2}{M_2^2} + \frac{(m_3^{D^2} c^2 + m_2^{D^2} s^2)^2}{M_3^2} + \frac{2(m_3^{D^2} - m_2^{D^2})^2 c^2 s^2 \cos 2\delta}{M_2 M_3}, \quad (31)$$

where $m_2 = m_{\nu_\mu}$ and $m_3 = m_{\nu_\tau}$ are the (positive) eigenvalues of m_ν , which are restricted by the small or large mixing angle MSW solution [18] of the solar neutrino puzzle and SuperKamiokande [17] respectively.

The $\mu - \tau$ mixing angle $\theta_{23} = \theta_{\mu\tau}$ lies [31] in the range

$$|\varphi - \theta^D| \leq \theta_{\mu\tau} \leq \varphi + \theta^D, \text{ for } \varphi + \theta^D \leq \pi/2, \quad (32)$$

where φ is the rotation angle which diagonalizes m_ν in the basis where m^D is diagonal and θ^D is the ‘Dirac’ mixing angle (i.e., the ‘unphysical’ mixing angle with zero Majorana masses for the right-handed neutrinos). We will assume, for simplicity, that θ^D is negligible, which implies $\theta_{\mu\tau} \simeq \varphi$. Also due to the presence of $SU(4)_c$ in G_{PS} , m_3^D coincides with the asymptotic value of the top quark mass. Taking renormalization effects into account, in the context of the MSSM with large $\tan \beta$, we find [31] $m_3^D = 110 - 120$ GeV. We also include the running of $\theta_{\mu\tau}$ from M_G to the electroweak scale [14].

In shifted hybrid inflation, for each κ and γ_3 , the $M_{2,3}$ are fixed. Taking $m_{2,3}$ and m_3^D also fixed in their allowed ranges, we are left with three undetermined parameters δ , θ and m_2^D which are restricted by four constraints: almost maximal $\nu_\mu - \nu_\tau$ mixing ($\sin^2 2\theta_{\mu\tau} \gtrsim 0.85$) from SuperKamiokande [17], the leptogenesis bound ($1.8 \times 10^{-10} \lesssim -n_L/s \lesssim 2.3 \times 10^{-10}$), and Eqs.(30) and (31). It is highly non-trivial that solutions satisfying all the above requirements can be found with natural κ ’s ($\sim 10^{-3}$) and m_2^D ’s of order 1 GeV (see last paragraph of this section). Typical solutions can be constructed, for instance, for $\kappa = 4 \times 10^{-3}$ (see Sec.2.1), which gives $m_{\text{infl}} \simeq 4.1 \times 10^{13}$ GeV, $M_2 \simeq 5.9 \times 10^{10}$ GeV and $M_3 \simeq 1.1 \times 10^{15}$ GeV (for $\gamma_3 = 0.5$). Taking, for

example, $m_{\nu_\mu} = 7.6 \times 10^{-3}$ eV, $m_{\nu_\tau} = 8 \times 10^{-2}$ eV and $m_3^D = 120$ GeV, we find $m_2^D \simeq 1.2$ GeV, $\sin^2 2\theta_{\mu\tau} \simeq 0.9$, $n_L/s \simeq -1.8 \times 10^{-10}$ and $\theta \simeq 0.016$ for $\delta \simeq -\pi/3$. Note that the m_{ν_μ} 's, for which solutions are found, turn out to be consistent with the large rather than the small mixing angle MSW mechanism.

In smooth hybrid inflation, we observe that no solutions can be found with $T_r \lesssim 10^9$ GeV, which is the gravitino constraint as usually quoted. We thus take $T_r = 10^{10}$ GeV, which is also perfectly acceptable provided [35] that the branching ratio of the gravitino to photons is somewhat smaller than unity and the gravitino mass is relatively large (\sim a few hundred GeV).

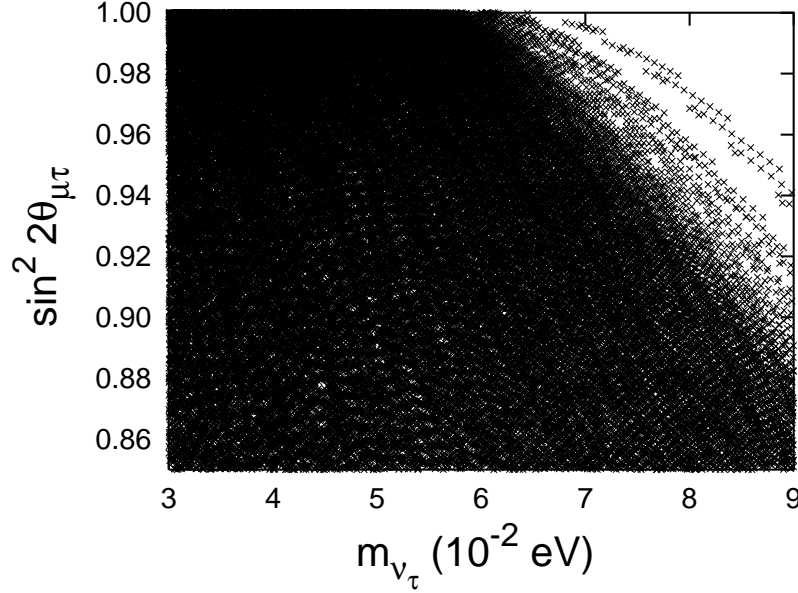


Figure 1. The scatter plot in the $m_{\nu_\tau} - \sin^2 2\theta_{\mu\tau}$ plane of the solutions which satisfy the low deuterium abundance constraint on the BAU, the restrictions from solar and atmospheric neutrino oscillations, and the $SU(4)_c$ invariance in the case of smooth hybrid inflation. We take $T_r = 10^{10}$ GeV, $\gamma_3 \approx 0.05 - 0.5$, $m_2^D \approx 0.8 - 2$ GeV and $m_3^D \approx 110 - 120$ GeV.

Our results are shown in Fig.1, where we plot solutions corresponding to $T_r = 10^{10}$ GeV and satisfying the leptogenesis constraint consistently with the neutrino oscillation data and the $SU(4)_c$ symmetry. The parameter γ_3 runs from 0.05 to 0.5, i.e., $M_3 \approx 1.86 \times 10^{14} - 1.86 \times 10^{15}$ GeV. The second inequality in Eq.(26) implies that $\gamma_3 \gtrsim 0.046$. However, no solutions are found for $\gamma_3 < 0.05$. Also, values of γ_3 higher than 0.5 do not allow solutions. The mass of the second heaviest right-handed neutrino $M_2 \approx 1.55 \times 10^{11}$ GeV, which clearly satisfies the first inequality in Eq.(26). The restrictions from $SU(4)_c$ invariance are expected to be more or less accurate only if applied to

the masses of the third family quarks and leptons. For the second family, they should hold only as order of magnitude relations. We thus restrict ourselves to values of m_2^D smaller than 2 GeV since much bigger m_2^D 's would violate strongly the $SU(4)_c$ symmetry (the value of m_2^D from exact $SU(4)_c$ is [31] about 0.23 GeV for MSSM spectrum with large $\tan\beta$). Moreover, we find that solutions exist only if $m_2^D \gtrsim 0.8$. So we take $m_2^D \approx 0.8 - 2$ GeV and, as required by $SU(4)_c$ invariance, $m_3^D \approx 110 - 120$ GeV. Also, the phase $\delta \approx (-\pi/8) - (-\pi/5)$ and the rotation angle $\theta \approx 0.01 - 0.03$ for solutions to appear. Note that δ 's close to 0 or $-\pi/2$ are excluded since they yield very small primordial lepton asymmetry.

4 Conclusions

We presented, in the context of the PS SUSY GUT model, two ‘natural’ extensions of hybrid inflation, which solve the cosmological monopole problem. These models reproduce the COBE measurements with ‘natural’ values of the parameters and a PS breaking scale close to (or equal with) the SUSY GUT scale. A PQ symmetry is used to generate the μ -term of MSSM and proton is practically stable. Inflation is followed by a successful ‘reheating’ satisfying the gravitino constraint on the ‘reheat’ temperature and generating the observed BAU via a primordial leptogenesis consistently with the requirements from solar and atmospheric neutrino oscillations and the $SU(4)_c$ symmetry.

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